5) Further increase of canting angle 1.9–2.7 deg results in quasi-periodic motion.

6) From 2.7 to 3.0 deg periodicity again appears.

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Decentralized Control of Expandingly Constructed Large Space Structures

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Introduction

IT is likely that future large space structures will be constructed by docking several modules together in orbit. Before docking, each module is stabilized by its own controller. However, the stability of the connected system is not guaranteed by the local controllers because they are designed for the isolated subsystems. In view of this, there is a need to design a stabilizing controller for the docking case when the plant properties drastically change. Decentralized control technology, which has the potential capability to stabilize interconnected systems with information constraints, seems to meet this requirement. A great deal of research has been carried out on this problem.[1–5] Their common framework is to obtain decentralized control system stability under the structural perturbations that result when modules are connected and disconnected in arbitrary ways. By this method, a set of local controllers achieves connective stability for space structures of arbitrary configuration. However, this is not necessarily the case in an actual construction scenario. It is usual that space structures are constructed by connecting new modules to already existing structures one after another. In this case, only the local controller of the new module should be designed so as to stabilize the connected system, without changing the existing system. This is called the expanding system problem.[6–8] This Note investigates stabilization during this type of construction. The controller design procedure is formulated in a μ synthesis framework, and a simple numerical example is used to illustrate its capability.

Problem Definition

Consider a space structure $\mathcal{S}$ to be constructed in orbit by connecting two modules $\mathcal{S}_1$ and $\mathcal{S}_2$. The mathematical models of $\mathcal{S}_i$ $(i = 1, 2)$ are described by modal equations:

$$M_i \ddot{p}_i + D_i \dot{p}_i + K_i p_i = L_i u_i, \qquad y_i = H_i p_i \tag{1}$$

where $p_i \in R^{N_i}$ is the modal coordinate vector and u_i and y_i are the control input and measurement output vectors, respectively. The diagonal matrices M_i, D_i, and K_i are the mass, damping, and stiffness in modal space. The connected system $\mathcal{S}$ after docking can be modeled by a direct finite element method (FEM) analysis or by a component mode synthesis.[9] Regardless of the approach taken, however, it is ultimately described by another modal equation as follows:

$$\begin{aligned} M\ddot{p} + D\dot{p} + Kp &= L_{.1}u_1 + L_{.2}u_2 \\ y_1 = H_{1.}p, &\qquad y_2 = H_{2.}p \end{aligned} \tag{2}$$

where $p \in R^N$ is the global modal coordinate after docking. The purpose of decentralized control synthesis is to obtain a pair of controllers

$$u_i = C_i(s)y_i \tag{3}$$

for $i = 1, 2$, which stabilizes $\mathcal{S}_1$, $\mathcal{S}_2$, and $\mathcal{S}$. We design the controllers by the following two steps: 1) design $C_1(s)$, which stabilizes $\mathcal{S}_1$, and 2) design $C_2(s)$, which stabilizes $\mathcal{S}_2$ and $\hat{\mathcal{S}}$ simultaneously, where $\hat{\mathcal{S}}$ is the closed-loop system of $\mathcal{S}$ with the controller $u_1 = C_1(s)y_1$. Because the first step is the ordinary vibration control problem, it will not be discussed in detail here. The design problem of $C_2(s)$ is investigated in the next section.

Controller Synthesis

We first derive two conditions for $C_2(s)$ to stabilize $\mathcal{S}_2$ and $\hat{\mathcal{S}}$ independently. It will then be shown that the latter condition includes the former under an assumption. The controller must be a reduced-order controller because the model order of Eqs. (1) and (2) is excessively large. This is an essential requirement for a vibration control problem. To investigate the controller stabilizing $\mathcal{S}_2$, Eq. (1) is rewritten in the frequency domain as $y_2 = P_2(s)u_2$. Then we separate $P_2(s)$ into a control model $Q_2(s)$ and a residual model $R_2(s)$ as $P_2(s) = Q_2(s) + R_2(s)$. We regard $Q_2(s)$ as the nominal model whose state-space realization is $Q_2(s) = C(sI - A)^{-1}B$, where $A \in R^{2n_2 \times 2n_2}$ and $n_2 \ll N_2$. Then $\mathcal{S}_2$ is equivalent to an extended system:

$$y_2 = Q_2(s)u_2 + w_2, \qquad z_2 = u_2$$

having the model error loop $w_2 = R_2(s)z_2$. Therefore, if $C_2(s)$ stabilizes $Q_2(s)$ and satisfies

$$\|R_2 T\|_\infty < 1, \qquad T(s) = C_2(s)[I - Q_2(s)C_2(s)]^{-1} \tag{4}$$

then $\mathcal{S}_2$ is robustly stabilized against $R_2(s)$. Next, the condition under which $C_2(s)$ stabilizes $\hat{\mathcal{S}}$ is considered. From Eq. (2), $\mathcal{S}$ is

$$y_1 = P_{11}(s)u_1 + P_{12}(s)u_2, \qquad y_2 = P_{21}(s)u_1 + P_{22}(s)u_2$$

in the frequency domain. Then the closed-loop system $\hat{\mathcal{S}}$ produced by $C_1(s)$ becomes

$$y_2 = \hat{P}_2(s)u_2, \qquad \hat{P}_2 = P_{21}C_1(I - P_{11}C_1)^{-1}P_{12} + P_{22} \tag{5}$$

We again describe $P_{ij}(s) = Q_{ij}(s) + R_{ij}(s)$ for $i, j = 1, 2$ so that $Q_{ij}(s)$ has the same number of modes as $Q_2(s)$; i.e., its realization is $Q_{ij}(s) = C_{q_i}(sI - A_q)^{-1}B_{q_j}$, where $A_q \in R^{2n_2 \times 2n_2}$. In the sequel,

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we make a generalized plant including the nominal model $Q_2(s)$ whose linear fractional transform with the model error is equivalent to Eq. (5). Then the controller $C_2(s)$ is designed such that it makes $Q_2(s)$ internally stable and robustly stable against model errors caused by the connection. For this purpose, the residual models are normalized as $R_{ij}(s) = \Delta_{r_{ij}}(s) W_r(s)$, $\bar{\sigma}\{\Delta_{r_{ij}}(j\omega)\} \le 1 \ \forall\omega$, where $W_r(s)$ is a stable proper real rational function. Then we consider the following extended system:

$$z_{r_1} = W_r(s) w_c, \qquad z_{r_2} = W_r(s) u_2$$
$$z_c = C_1(s)\big[w_{r_1} + Q_{11}(s) w_c + Q_{12}(s) u_2\big] \tag{6}$$
$$y_2 = w_{r_2} + Q_{21}(s) w_c + Q_{22}(s) u_2$$

It can be shown that the upper linear transform of Eq. (6) with

$$w_{r_1} = \Delta_{r_{11}} z_{r_1} + \Delta_{r_{12}} z_{r_2}$$
$$w_{r_2} = \Delta_{r_{21}} z_{r_1} + \Delta_{r_{22}} z_{r_2}, \qquad w_c = z_c \tag{7}$$

yields Eq. (5), by eliminating fictitious inputs w_{r_1}, w_{r_2}, and w_c and outputs z_{r_1}, z_{r_2}, and z_c from Eqs. (6) and (7). Next, we describe $Q_{22}(s)$ by $Q_2(s)$ and parameter errors. The real parameter perturbations are described as $A_q = A + E_a \Delta_a F_a$, $B_{q_2} = B + E_b \Delta_b F_b$, and $C_{q_2} = C + E_c \Delta_c F_c$, where $|\Delta_i| \le 1$ for $i = a, b, c$. By introducing additional fictitious input w_q and output z_q, we can obtain the following generalized plant from Eq. (6):

$$z_q = F\Phi^{-1}(s) E w_q + F\Phi^{-1}(s) B_{q_1} w_c + [F\Phi^{-1}(s) B + \bar{F}] u_2$$
$$z_{r_1} = W_r(s) w_c, \qquad z_{r_2} = W_r(s) u_2$$
$$z_c = C_1(s) C_{q_1} \Phi^{-1}(s) E w_q + C_1(s) w_{r_1} \tag{8}$$
$$+ C_1(s) C_{q_1} \Phi^{-1}(s) B_{q_1} w_c + C_1(s) C_{q_1} \Phi^{-1}(s) B u_2$$
$$y_2 = [C\Phi^{-1}(s) E + \bar{E}] w_q + w_{r_2} + C\Phi^{-1}(s) B_{q_1} w_c + Q_2(s) u_2$$

where $\Phi(s) = sI - A$, $E = [E_a \ 0 \ E_c]$, $\bar{E} = [0 \ E_b \ 0]$, $F^T = [F_a^T \ 0 \ F_c^T]$, and $\bar{F}^T = [0 \ F_b^T \ 0]$. Equation (8) is compactly expressed as

$$\bar{z} = \bar{Q}_{11}(s)\bar{w} + \bar{Q}_{12}(s) u_2, \qquad y_2 = \bar{Q}_{21}(s)\bar{w} + Q_2(s) u_2 \tag{9}$$

Note again that the upper linear fractional transform of Eq. (9) with the model error $\bar{w} = \bar{\Delta}\bar{z}$ becomes Eq. (5). Therefore, if $C_2(s)$ stabilizes $Q_2(s)$ and satisfies

$$\mu_{\bar{\Delta}}(T_q) < 1$$
$$T_q(s) = \bar{Q}_{11}(s) + \bar{Q}_{12}(s) C_2(s)[I - Q_2(s) C_2(s)]^{-1} \bar{Q}_{21}(s) \tag{10}$$

then $\hat{\mathcal{S}}$ is stabilized. From the preceding discussion, we have two conditions that $C_2(s)$ should satisfy. Equation (4) defines the class of $C_2(s)$ stabilizing $\mathcal{S}_2$, and Eq. (10) defines that stabilizing $\hat{\mathcal{S}}$. It can be proved as follows that the latter class includes the former.

Lemma: When $C_2(s)$ satisfies $\mu_{\bar{\Delta}}(T_q) < 1$, if $\bar{\sigma}\{R_2\} < \underline{\sigma}\{W_r\} \ \forall\omega$, then it also satisfies $\|R_2 T\|_\infty < 1$.

Proof: Because the controller $C_2(s)$ makes $Q_2(s)$ stable as well as robustly stable against the perturbation $\bar{\Delta}$, $\det(I - Q_2 C_2) \ne 0$, and $\det(I - \bar{\Delta} T_q) \ne 0$, $\forall\bar{\Delta}$ and $\forall\omega$ hold. Therefore, even when $\bar{\Delta} = \mathrm{diag}\{0 \ 0 \ \Delta_r\}$, the closed-loop system is stable, that is,

$$\det(I - \Delta_r T_{q_{rr}}) \ne 0, \quad \forall\omega, \forall\Delta_r$$
$$\Leftrightarrow \det[I - \Delta_{r22} W_r C_2 (I - Q_2 C_2)^{-1}] \ne 0, \quad \forall\omega, \forall\Delta_{r22}$$
$$\Leftrightarrow \bar{\sigma}\{W_r T\} < 1, \quad \forall\omega$$
$$\Rightarrow \bar{\sigma}\{T\} < \bar{\sigma}\left\{W_r^{-1}\right\}, \quad \forall\omega$$

where $T_{q_{rr}}(s)$ is the submatrix of $T_q(s)$ corresponding from the input w_r to the output z_r. Because $\bar{\sigma}\{W_r^{-1}\} = 1/\underline{\sigma}\{W_r\}$, if $\bar{\sigma}\{R_2\} \le \underline{\sigma}\{W_r\}$, then $\bar{\sigma}\{R_2 T\} < 1$. □

In summary, what we intend to design is a controller $C_2(s)$ satisfying Eq. (10), under the assumption given in the preceding Lemma. Obviously, such a controller can be obtained using the standard μ synthesis algorithm.[10]

Numerical Example

We consider the case of two beams being connected as shown in Fig. 1. This discussion is restricted to rotational motion around the normal axis coupled with bending vibration. Both beams are assumed to have the same length, $L = 5$ m, and the same structural properties, $EI = 10^4$ N/m^2 and $\rho A = 1$ kg/m. The modal parameters are obtained from a 10-element FEM model to yield two rigid and five vibration modes. By eliminating a rigid translational mode from the modal equation, the control system of each beam having a collocated torque actuator and rotation sensor at the tip is stabilizable and detectable. First, controller $C_1(s)$ is designed as a mixed sensitivity H^∞ problem by considering the rigid-body mode as the control mode. The robust stability degree assignment method[11] is applied to achieve the prescribed closed-loop poles location. As the next step, we design $C_2(s)$. By selecting only the rigid-body mode being controlled, $Q_2(s) = C(sI - A)^{-1}B$ is a single-input/single-output system with $A \in R^{2\times 2}$. All the vibration modes are included in $R(s)$ in this numerical study. The weighting function is a second-order transfer function $W_r(s) = (f_1 s + f_2)/(s^2 + e_1 s + e_2)$. The parameters f_i and e_i are selected as $f_1 = 0.84$, $f_2 = 1.44$, $e_1 = 44.3$, and $e_2 = 1000$ to satisfy the condition in the Lemma. For μ synthesis, the controller $C_2(s)$ satisfying Eq. (10) is calculated by the iteration based on the inequality

$$\min_{C_2} \inf_{D} \left\| D T_q D^{-1} \right\|_\infty < 1$$

We performed the iteration three times because no significant improvement was observed at the third iteration. Figure 2 shows the impulse disturbance responses of rotation angles and control inputs before and after docking. The controller resulting from the first iteration is shown by dotted lines and that from the second iteration by solid lines. In the two right-hand graphs of Fig. 2, responses of y_1 and y_2 seem to be duplicated. They are the closed-loop behavior of rotational rigid-body modes. The vibration modes are quickly damped out as shown by the responses of u_1 and u_2. Obviously, the performance of the second iteration is better than that of the first iteration.

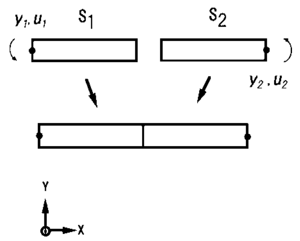
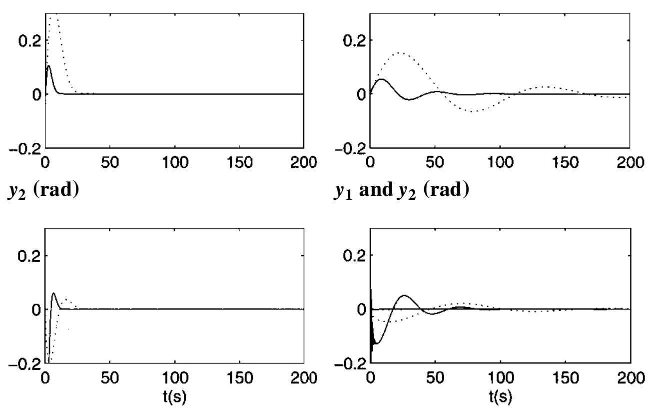


Fig. 1 Configuration of two-beam connection.



y_2 **(rad)** y_1 **and** y_2 **(rad)**

u_2 **(Nm)** u_1 **and** u_2 **(Nm)**

Fig. 2 Impulse disturbance responses: before docking (left) and after docking (right) for controller of · · · · ·, first and ——, second iteration.

Conclusion

We have proposed a controller design procedure for stabilizing expandingly constructed space structures in a robust control framework. By an example of two flexible beams, its validity has been shown. Although only a stabilizing problem is discussed herein, control performance specifications can be incorporated by the similar formulation. When a large space structure is built by connecting several modules, a set of local controllers can be designed by repeating the procedure sequentially.

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